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Implementing Deutsch-Jozsa Algorithm with Superconducting Quantum Interference Devices in Cavity QED

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Abstract We propose a scheme for implementing the Deutsch-Jozsa (DJ) algorithm with superconducting quantum interference devices (SQUIDs) in cavity quantum electrodynamics (QED). The required controlled-NOT (CNOT) operations can be easily realized based on the resonant interaction of SQUIDs with a single-mode high-Q cavity. The scheme has the advantages of being simple, scalable and feasible in the experimental realization and further utilization.

Keywords DJ algorithm · SQUIDs · Cavity QED

The principles of quantum mechanics provided many important and interesting applications in the field of quantum computing and quantum information processing [1–6] in the last decade. Since the discovery of quantum mechanics, people have paid much attention to quantum computers [7], which can efficiently perform some tasks that are not feasible on a classical computer using quantum parallelism and interference effect, such as factoring problem [8], phase estimation problem [9], search problem [10], hidden subgroup problem [11, 12], and so on. Shor's algorithm [8] for factorizing a large composite number can be achieved in polynomial time, which provides an exponential speedup over the best known classical algorithm. The Grover algorithm [10] gives a quadratic speedup over the most efficiently classical search algorithms for searching a marked item from an unordered database of size $N = 2^n$. The DJ algorithm [13], which can distinguish whether an unknown function is constant or balanced, is also a very simple and important quantum algorithm and provides an exponential speedup with respect to classical algorithm. Up to now, the DJ algorithm has been widely studied in both theoretical and experimental aspects.

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The DJ algorithm can be briefly described as follows. Consider a Boolean function $f(x)$ defined from a N-bit domain space to a one-bit space: $f(x):\{0,1\}^N \to \{0,1\}$. If for half of all the possible 2^n inputs, the function values are equal to 0, and 1 for the other half, then the function is called balanced function. Otherwise, it is called constant function if the function values are constant for all 2^n inputs. With a classical algorithm, at worst, it requires $2^n/2+1$ evaluations of the function $f(x)$ for different values of x to determine whether the function is constant or balanced, while the DJ algorithm only requires a single evaluation of the function $f(x)$ with the following steps.

(i) The qubits in the first and second registers are initially prepared in the state $|\varphi\rangle_0 = |00\cdots 0_n\rangle_1 \otimes |1\rangle_2$.
(ii) Apply the Walsh-Hadamard transform defined as $|x\rangle \to 1/\sqrt{2}\sum_{y=0,1}(-1)^{x\cdot y}|y\rangle$ ($x \in \{0,1\}$) to each of the qubits.
(iii) Perform the f-controlled phase shift U_f

$$|x\rangle_1|y\rangle_2 \xrightarrow{U_f} |x\rangle_1|y\oplus f(x)\rangle_2, \quad x,y\in\{0,1\}^n, \tag{1}$$

where $\oplus$ denotes addition modulo 2.

(iv) Perform the Walsh-Hadamard transform again. Then the resulting state is

$$|\varphi\rangle_r = \sum_{x=0}^{2^n-1}\sum_{y=0}^{2^n-1}\frac{(-1)^{x\cdot y+f(x)}}{2^n}|y\rangle_1 \otimes \frac{1}{\sqrt{2}}\left[(|0\rangle_2 - |1\rangle_2)\right]. \tag{2}$$

(v) Measure the resulting state in first register. If the result is $|00\ldots 0_n\rangle_1$ for the state of all the qubits, then the function $f(x)$ is constant; otherwise, the function $f(x)$ is balanced.

In the simple case of two qubits system consisting of one query qubit 1 and an auxiliary working qubit 2, on which we will focus from now on, the initial state, after steps (i–iii) mentioned above, becomes

$$|\psi\rangle = \frac{1}{2}[(-1)^{f(0)}|0\rangle_1 + (-1)^{f(1)}|1\rangle_1]\otimes(|0\rangle_2 - |1\rangle_2). \tag{3}$$

There will exist four possible transformations for U_{f_n} ($n=1,2,3,4$) shown in Table 1. After applying the Walsh-Hadamard transform to qubit 1, the state of qubit 1 will be in $|0\rangle$ corresponding to U_{f_1} and U_{f_2} operations, while for U_{f_3} and U_{f_4} operations, qubit 1 will be in the state $|1\rangle$. Thus a single measurement on the qubit 1 can determine whether the function is constant or balanced.

Table 1 Different U_{f_n} transformations corresponding to the possible values that the function $f_n(x)$ can hold and their characterizations for the two-qubit Deutsch-Jozsa algorithm

Input	Constant		Balanced	
x	$f_1(x)$	$f_2(x)$	$f_3(x)$	$f_4(x)$
0	0	1	0	1
1	0	1	1	0
Corresponding operations	U_{f_1}	U_{f_2}	U_{f_3}	U_{f_4}

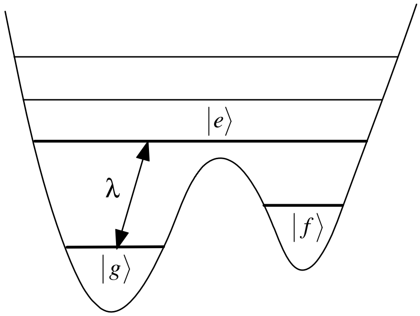

Fig. 1 Level diagram of a Λ-type SQUID with three lowest levels denoted by $|g\rangle$, $|f\rangle$, and $|e\rangle$

Up to now, many schemes have been proposed to implement the DJ algorithm in ion trap system [14], linear optical system [15], cavity QED system [16–18] and nuclear magnetic resonance system [19, 20]. Recently, Zheng [18] proposed a scheme to implement two-qubit DJ algorithm based on resonant interaction of atoms with a cavity mode. In the scheme, the state of the cavity mode was used as the auxiliary working qubit after the first atom interacted with the cavity mode resonantly, which makes the scheme sensitive to the cavity decay and the thermal field, leading to the practical experiment difficult to be scalable. Dasgupta et al. [21] proposed an optical scheme for implementing two-qubit DJ algorithm using light shifts and atomic ensembles. However, the implementations of required one- and two-qubit operations are very complex. Furthermore, there is no report about the realization of the two schemes in experiment yet. At present, SQUIDs have been paid to much attention and they have become one of the most promising candidates for physical implementations of quantum computation and other quantum information processing since the superconducting qubits have relatively long decoherence times and can be easily scaled up. The SQUIDs-cavity system provides a new way for production of nonclassical microwave source and quantum communication. Therefore, one can easily realize physical implementations of quantum computation and quantum information processing using SQUID qubits in cavity QED. Recent advance in SQUIDs has opened wide prospects for building up superconducting quantum computers and information processors. In this paper, we propose a simple and experimentally realizable scheme for implementing the two-qubit DJ algorithm with SQUIDs based on cavity QED. The scheme has the following advantages: (i) qubit definitions are the same for the SQUIDs, which makes the experiment easier; (ii) the time required to implement the DJ algorithm is very short due to the resonant interaction; (iii) the coupling strength of SQUIDs with the cavity mode can be different, thus both of the SQUIDs are not required to be identical and to be located in the exact placement; (iii) the interaction between the SQUIDs can be canceled since each time only one SQUIDs interacts with the cavity mode. Based on the presently available techniques for SQUIDs and cavity QED, our scheme might be experimentally realizable.

Consider a Λ-type three-level SQUID (Fig. 1) interacting with a high-Q cavity field. If the cavity mode is resonant with the $|g\rangle \leftrightarrow |e\rangle$ transition while far off resonant with the $|f\rangle \leftrightarrow |e\rangle$ transition and the $|g\rangle \leftrightarrow |f\rangle$ transition of the SQUID, the time evolution of the state can be written as [22]

$$\begin{aligned} |g\rangle|n\rangle &\rightarrow \cos(\sqrt{n}\lambda t)|g\rangle|n\rangle - i\sin(\sqrt{n}\lambda t)|e\rangle|n-1\rangle, \\ |e\rangle|n\rangle &\rightarrow \cos(\sqrt{n+1}\lambda t)|e\rangle|n\rangle - i\sin(\sqrt{n+1}\lambda t)|g\rangle|n+1\rangle, \end{aligned} \tag{4}$$

here λ is the effective coupling constant and n is the photon number state of the cavity.

To implement the two-qubit DJ algorithm, we first prepare the two SQUIDs in the state

$$|\Psi\rangle_1 = \frac{1}{\sqrt{2}}(|g\rangle_1 + |e\rangle_1) \otimes |g\rangle_2. \tag{5}$$

Here we let the first SQUID serve as the query qubit and the second SQUID as the auxiliary working qubit. In the new rotated basis for the SQUID 2, the state can be rewritten as

$$|\Psi\rangle_1 = \frac{1}{2}(|g\rangle_1 + |e\rangle_1) \otimes (|+\rangle_{2,r} - |-\rangle_{2,r}), \tag{6}$$

where

$$\begin{aligned} |+\rangle_{2,r} &= \frac{1}{\sqrt{2}}(|f\rangle_2 + |g\rangle_2), \\ |-\rangle_{2,r} &= \frac{1}{\sqrt{2}}(|f\rangle_2 - |g\rangle_2). \end{aligned} \tag{7}$$

Next the U_{f_n} operations, which are the unitary operators corresponding to each function $f_n(x)$, are applied to state $|\Psi\rangle_1$. In the following we will show how to realize the U_{f_n} operations in detail. Initially, the two SQUIDs are embedded in a high-Q cavity and the cavity mode is initially in the vacuum state $|0\rangle_c$.

U_{f_1} operation. This is an identity operation. In this case the SQUIDs can be tuned far off resonant with the cavity mode and thus the SQUID-cavity evolution is freezing. Thus the system remains in the state of $|\Psi\rangle_1$.

U_{f_2} operation. Firstly, turn-off SQUID 2 (i.e., the SQUID 2 does not interact with the cavity field), and let the $|g\rangle \leftrightarrow |e\rangle$ transition of SQUID 1 resonantly interact with the cavity mode for an interaction time $t_1 = 3\pi/(2\lambda)$. Secondly, turn-off SQUID 1, and let the $|g\rangle \leftrightarrow |e\rangle$ transition of SQUID 2 resonantly interact with the cavity mode for an interaction time $t_2 = \pi/\lambda$. Thirdly, turn-off SQUID 2 again, and let the $|g\rangle \leftrightarrow |e\rangle$ transition of SQUID 1 resonantly interact with the cavity mode for an interaction time $t_3 = \pi/(2\lambda)$. The above operations realize a two-qubit CNOT operation acting on the state $|\Psi\rangle_1$ and are summarized as below

$$\begin{array}{ccccccc} |g\rangle_1|+\rangle_{2,r}|0\rangle_c & & |g\rangle_1|+\rangle_{2,r}|0\rangle_c & & |g\rangle_1|+\rangle_{2,r}|0\rangle_c & & |g\rangle_1|+\rangle_{2,r}|0\rangle_c \\ |g\rangle_1|-\rangle_{2,r}|0\rangle_c & \xrightarrow{\text{firstly}} & |g\rangle_1|-\rangle_{2,r}|0\rangle_c & \xrightarrow{\text{secondly}} & |g\rangle_1|-\rangle_{2,r}|0\rangle_c & \xrightarrow{\text{thirdly}} & |g\rangle_1|-\rangle_{2,r}|0\rangle_c \\ |e\rangle_1|+\rangle_{2,r}|0\rangle_c & & i|g\rangle_1|+\rangle_{2,r}|1\rangle_c & & i|g\rangle_1|-\rangle_{2,r}|1\rangle_c & & |e\rangle_1|-\rangle_{2,r}|0\rangle_c \\ |e\rangle_1|-\rangle_{2,r}|0\rangle_c & & i|g\rangle_1|-\rangle_{2,r}|1\rangle_c & & i|g\rangle_1|+\rangle_{2,r}|1\rangle_c & & |e\rangle_1|+\rangle_{2,r}|0\rangle_c \end{array}. \tag{8}$$

Next we perform the single-qubit transformation $|g\rangle_1 \rightarrow |e\rangle_1$ and $|e\rangle_1 \rightarrow -|g\rangle_1$ on the SQUID 1 by using a π-Ramsey pulse. Then we repeat the CNOT operation of (8) and perform the transform $|g\rangle_1 \rightarrow -|e\rangle_1$ and $|e\rangle_1 \rightarrow |g\rangle_1$ by using another π-Ramsey pulse with a phase difference π relative to the first Ramsey pulse. Finally, we obtain

$$|\Psi\rangle_2 = \frac{1}{2}(-|g\rangle_1 - |e\rangle_1) \otimes (|+\rangle_{2,r} - |-\rangle_{2,r}). \tag{9}$$

U_{f_3} operation. We perform the CNOT operation of (8), obtaining

$$|\Psi\rangle_3 = \frac{1}{2}(|g\rangle_1 - |e\rangle_1) \otimes (|+\rangle_{2,r} - |-\rangle_{2,r}). \tag{10}$$

Table 2 Experimental parameters for SQUID and cavity revealed in [23, 24]. Where λ_c is the wavelength of the cavity mode, ΔS is the area of the SQUID, and L is the loop inductance

Cavity	$\Delta S = 45 \times 45\ \mu\mathrm{m}^2$	$\gamma_e^{-1} \sim 2.5\ \mu\mathrm{s}$	$\gamma_f^{-1} \sim 100\ \mu\mathrm{s}$
	$Q \sim 1 \times 10^6$	$V \sim 1 \times 7.2\ \mathrm{mm}^3$	$\lambda_c = 7.2$ mm
	$v_c = 41.7$ GHz		
SQUID	$L = 36$ pH	$v_{eg} = 41.7$ GHz	$v_{ef} = 33.3$ GHz

U_{f_4} *operation*. We first perform the single-qubit transformation $|g\rangle_1 \to |e\rangle_1$ and $|e\rangle_1 \to -|g\rangle_1$ on the SQUID 1. Then we perform the CNOT operation of (8). At last, we perform the single-qubit transformation $|g\rangle_1 \to -|e\rangle_1$ and $|e\rangle_1 \to |g\rangle_1$. The resulting state is given by

$$|\Psi\rangle_4 = \frac{1}{2}(-|g\rangle_1 + |e\rangle_1) \otimes (|+\rangle_{2,r} - |-\rangle_{2,r}). \tag{11}$$

In this way the U_{f_n} operations are realized. After performing the Walsh-Hadamard transform on the SQUID 1, we make a single-qubit measurement. If the measurement result is $|g\rangle_1$, the function $f(x)$ is constant, while for the measurement result $|e\rangle_1$, the function $f(x)$ is balanced.

We briefly discuss the experimental feasibility of our scheme. The total time required for implementing the DJ algorithm should satisfy $t \ll \gamma_e^{-1},\ \kappa^{-1}$. Here $\kappa^{-1} \sim 3.8 \times 10^{-6}$ s is the photon lifetime of the cavity and $\gamma_e^{-1} \sim 2.5 \times 10^{-6}$ s is the spontaneous emission lifetime of the level $|e\rangle$ [23, 24]. The effective SQUID-cavity coupling strength can be easily obtained by calculating the parameters shown in Table 2, namely, $\lambda \sim 2.5 \times 10^9\ \mathrm{s}^{-1}$. Direct calculation shows that the times required to realize U_{f_n} operations are $t_{U_{f_2}} = 50$ ns and $t_{U_{f_3}} = t_{U_{f_4}} = 25$ ns, much shorter than γ_e^{-1} and κ^{-1}. In addition, in the scheme we require that the irrelevant SQUIDs do not interact with the cavity (pulse) during the SQUID-cavity (SQUID-pulse) resonant interaction and the cavity mode requires to be not excited when the SQUIDs interact with the pulse resonantly. For satisfying the above conditions, in principle, we can change the level spacings of SQUIDs by varying the external flux Φ_x or the critical current I_c [25]. Therefore, the proposed scheme might be experimentally realizable based on the presently available techniques in cavity QED.

In summary, we have proposed a scheme for implementing the two-qubit DJ algorithm based on the resonant interaction of SQUIDs with a single-mode cavity. The required operations can be easily realized and the time required for implementing the DJ algorithm is very short. The implementation of the scheme in experiment would open a wide prospects for more complicated quantum computation with SQUIDs in cavity QED.

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